Uncertainty in pressure-based velocity probes

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Consider a conventional Pitot-static probe operating in low subsonic conditions, using a differential pressure transducer. The ith reading of the transducer will be related to the local velocity according to the relationship

$$(P_0 - P_i) = \frac{1}{2}\rho U_i^2, \tag{1}$$

where P is the local static pressure, P_0 is the stagnation pressure (assumed invariant, so that the difference $P_0 - P_i$ may be directly read from a differential pressure transducer between the static and stagnation lines), and U is the local mean velocity. If we then consider a case when the pressure changes from some state 1 to some other state 2, (1) may be re-written as

$$\left(P_0 - P_2\right) - \left(P_0 - P_1\right) = \frac{1}{2}\rho U_2^2 - \frac{1}{2}\rho U_1^2. \tag{2}$$

The stagnation pressure may then be eliminated, and (2) re-arranged to yield

$$P_1 - P_2 = \frac{1}{2}\rho\left(U_2^2 - U_1^2\right). \tag{3}$$

The difference on the right-hand side may then be expanded, so that

$$\begin{aligned} \Delta P &= \frac{1}{2}\rho\Delta U\left(U_2 + U_1\right) \\ &= \frac{1}{2}\rho\Delta U\left(U_2 - U_1 + 2U_1\right) \end{aligned} \tag{4}$$

where the Δ operator represents the signed difference in any quantity moving from state 1 to state 2. Simplifying,

$$\Delta P = \frac{1}{2}\rho\Delta U\left(\Delta U + 2U_1\right). \tag{5}$$

If it is assumed that the change in pressure is due to some small perturbation, then it is reasonable to expect that $U_2 \approx U_1 = U$, and the differences ΔP

and ΔU may be expressed instead as dimensional uncertainties ϵ_P and ϵ_U, respectively. Then, (5) may be expressed as

$$\epsilon_P = \frac{1}{2}\rho\left(\epsilon_U^2 + 2U\epsilon_U\right). \tag{6}$$

Normalizing, this yields

$$\frac{\epsilon_P}{P_0 - P} = \frac{2\epsilon_P}{\rho U^2} = \left(\frac{\epsilon_U^2}{U^2} + 2\frac{\epsilon_U}{U}\right). \tag{7}$$

Alternatively, the uncertainty in velocity may be expressed as a function of the uncertainty in pressure by isolating ϵ_U in the above, as

$$\frac{\epsilon_U}{U} = \sqrt{\frac{\epsilon_P}{P_0 - P} + 1} - 1 \tag{8}$$

The relationships shown in (7) and (8) is illustrated graphically in figure 1.

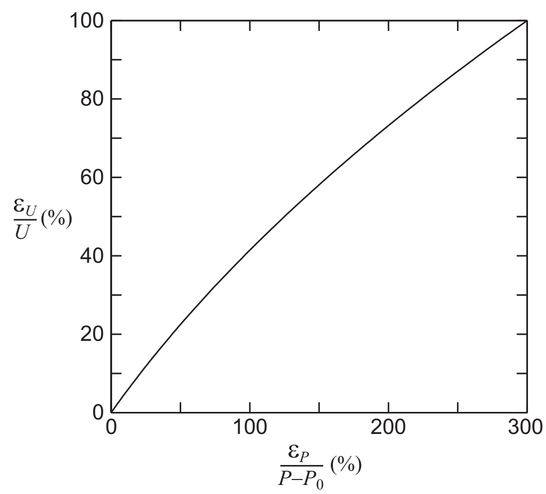

Figure 1: Relationship between the relative uncertainty in pressure and relative uncertainty in velocity for a standard Pitot-static probe.

In practice, because of the nonlinearity in the relationship between pressure and velocity, the uncertainty in velocity will always be greater than the uncertainty in pressure- and this difference can be very large at small speeds. For example, a differential pressure sensor having a 250 Pa full-scale range, when connected to a standard Pitot-static probe, can detect a maximum velocity of about 20 m/s. If the pressure sensor has an uncertainty of ±0.5% FS (which is typical), then the uncertainty in speed at 20 m/s is about ± 5 cm/s. However, the uncertainty can be as high as 1.5 m/s at when the speed becomes small.